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Local-global properties for semistar operations. (English. English summary)

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Let D be an integral domain with quotient field K. A semistar operation $*$ on D, introduced in 1994 by A. Okabe and R. Matsuda [Math. J. Toyama Univ. **17** (1994), 1–21; MR1311837 (95k:13027)], is a closure operation on the set $\overline{F}(D)$ of nonzero D-submodules of K that satisfies $(xE)^* = xE^*$ for each $0 \neq x \in K$ and each $E \in \overline{F}(D)$. For any $P \in \mathrm{Spec}(D)$, the semistar operation $*^{D_P}$, denoted simply by $*_P$, on the localization D_P is obtained from $*$ by $E^{*_P} := E^*$ for each $E \in \overline{F}(D_P)$ $(\subseteq \overline{F}(D))$. On the other hand, let Θ be a nonempty subset of $\mathrm{Spec}(D)$ and let $\{*_P \mid P \in \Theta\}$ be a family of semistar operations, where $*_P$ is a semistar operation on D_P. Then the authors define $\wedge (:= \wedge_\Theta)$ as the semistar operation on D given by $E^\wedge := \bigcap\{(ED_P)^{*_P} \mid P \in \Theta\}$ for each $E \in \overline{F}(D)$.

In this paper, the authors conduct a local-global study of semistar operations by exploring conditions under which certain properties relevant to semistar operations transfer by way of the above induced semistar operations. In particular, they show that the finite type, spectral, stable, a.b., and e.a.b. properties on a semistar operation $*$ each directly transfer to the induced semistar operation $*_P$ for any $P \in \mathrm{Spec}(D)$. The authors also demonstrate that D is a Prüfer $\star$-multiplication domain if and only if each of its localizations D_P is a Prüfer $*_P$-multiplication domain. In addition, the authors provide a nice study of the relationship between the Nagata rings $\mathrm{Na}(D,*)$ and $\mathrm{Na}(D_P,*_P)$, and the relationship between the Kronecker function rings $\mathrm{Kr}(D,*)$ and $\mathrm{Kr}(D_P,*_P)$. They show, amongst other results, that $\mathrm{Na}(D,*) = \bigcap\{\mathrm{Na}(D_P,*_P) \mid P \in \mathrm{Spec}(D)\}$ and $\mathrm{Kr}(D,*) = \bigcap\{\mathrm{Kr}(D_P,*_P) \mid P \in \mathrm{Spec}(D)\}$. In the final section of the paper, the authors demonstrate that if $D = \bigcap\{D_P \mid P \in \Theta\}$ has finite character and each $*_P$, $P \in \Theta$, is of finite type (respectively, stable), then $\wedge_\Theta$ inherits the finite type (respectively, stable) property. They also show that if each $*_P$, $P \in \Theta$, is a spectral e.a.b. (respectively, a.b.) semistar operation, then $\wedge_\Theta$ inherits the e.a.b. (respectively, a.b.) property under suitable conditions on $\{*_P \mid P \in \Theta\}$. *Andrew J. Hetzel* (1-LA2)

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