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On the star class group of a pullback. (English summary)
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Let $*$ be a star operation on an integral domain R. Then $\mathrm{Cl}_*(R)$, the $*$-class group of R, is the abelian group of $*$-invertible (fractional) $*$-ideals of R under $*$-multiplication modulo its subgroup of principal (fractional) ideals. In this paper, the authors investigate the case where R arises as a pullback. Let T be an integral domain, M a nonzero maximal ideal of T, D a proper subring of $k = T/M$ with quotient field k, $\varphi: T \to k$ the natural projection, and $R = \varphi^{-1}(D)$. They show that a star operation $*$ on R induces star operations $*_\varphi$ on D and $(*)_T$ on T, and that if $*$ has finite type and the natural map $U(T) \to k^*/U(D)$ is surjective, then these star class groups are related by a split exact sequence $0 \to \mathrm{Cl}_{*_\varphi}(D) \to \mathrm{Cl}_*(R) \to \mathrm{Cl}_{(*)_T}(T) \to 0$. Special emphasis is given to the case where $*$ is the t-operation on R.

Reviewed by *David F. Anderson*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.