Soluzioni Prova scritta di AM4 del 13/1/2004
(Appello A e recupero Esoneri)

1)

$$\int \sin^2 t \sin^2 \omega t\, dt = \frac{t}{4} - \frac{1}{8}\sin 2t - \frac{1}{8\omega}\sin 2\omega t + \frac{1}{8(1-\omega^2)}\big(\sin 2t \cos 2\omega t - \omega \cos 2t \sin 2\omega t\big).$$

$$\lim_{\omega\to\infty}\int_0^1 \sin^2 t \sin^2 \omega t\, dt = \frac{1}{4} - \frac{1}{8}\,\mathrm{sen}\, 2.$$

2) $\displaystyle\int \frac{x^2}{\sqrt{(x^2-1)^3}}\,dx = \log\big(x+\sqrt{x^2-1}\big) - \frac{x}{\sqrt{x^2-1}}.$

3)

$$\int_S f\,d\sigma = \sqrt{3}\Big[\frac{1}{12}(1+4u^2)^{3/2} + \frac{1}{32}\Big(u\sqrt{1+4u^2}(1+8u^2) - \frac{1}{2}\ln\big(2u+\sqrt{1+4u^2}\big)\Big) + \Big(\sqrt{3}-\frac{1}{2}\Big)\Big(u\sqrt{1+4u^2} + \frac{1}{2}\ln\big(2u+\sqrt{1+4u^2}\big)\Big)\Big]_0^1.$$

7) (i) $\alpha > -2$ (essendo $x^\alpha\,\mathrm{sen}\,x$ integrabile su $(0,\pi)$ se e solo se $\alpha > -2$).
(ii) Se $\hat{F}_{\alpha,\pm n} = (a_{\alpha,n} \mp b_{\alpha,n})/2$ si ha che $a_{\alpha,n} = 0$ per ogni $n \geq 0$ e

$$b_{1,1} = \frac{\pi}{2}\,, \qquad b_{1,n} = \frac{(-1)^n+1}{\pi}\,\frac{-4n}{(n^2-1)^2}\,, \qquad \forall\, n \geq 2\,.$$

(iii) Dal lemma di Riemann-Lebesgue segue che $b_{-1,n} \sim 1/n$ e $b_{100,n} \sim 1/n^3$. Più precisamente:

$$\frac{2}{\pi}b_{-1,n} = \frac{1}{n} + \frac{1}{n}\int_0^\pi \cos(nx)\Big(\frac{\mathrm{sen}\,x}{x}\Big)' = \frac{1}{n} + o\Big(\frac{1}{n}\Big);$$

$$\frac{2}{\pi}b_{100,n} = \frac{(-1)^n}{n^3}\,200\,\pi^{99} + \frac{1}{n^3}\int_0^\pi \cos(nx)\Big(x^{100}\,\mathrm{sen}\,x\Big)^{(3)} = \frac{(-1)^n}{n^3}\,200\,\pi^{99} + o\Big(\frac{1}{n^3}\Big)$$

8) (i) $\displaystyle\hat{f}(\xi) = \frac{-i}{\sqrt{\pi}}\Big(\frac{\mathrm{sen}\,(1-\xi)}{1-\xi} - \frac{\mathrm{sen}\,(1+\xi)}{1+\xi}\Big).$

(ii) $\displaystyle u(x,t) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-\xi^4 t}\, e^{ix\xi}\, \hat{f}(\xi)d\xi.$