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MANIFOLD DESTINY

A legendary problem and the battle over who solved it.

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On the evening of June 20th, several hundred physicists, including a Nobel laureate, assembled in an auditorium at the Friendship Hotel in Beijing for a lecture by the Chinese mathematician Shing-Tung Yau. In the late nineteen-seventies, when Yau was in his twenties, he had made a series of breakthroughs that helped launch the string-theory revolution in physics and earned him, in addition to a Fields Medal—the most coveted award in mathematics—a reputation in both disciplines as a thinker of unrivalled technical power.

Yau had since become a professor of mathematics at Harvard and the director of mathematics institutes in Beijing and Hong Kong, dividing his time between the United States and China. His lecture at the Friendship Hotel was part of an international conference on string theory, which he had organized with the support of the Chinese government, in part to promote the country's recent advances in theoretical physics. (More than six thousand students attended the keynote address, which was delivered by Yau's close friend Stephen Hawking, in the Great Hall of the People.) The subject of Yau's talk was something that few in his audience knew much about: the Poincaré conjecture, a century-old conundrum about the characteristics of three-dimensional spheres, which, because it has important implications for mathematics and cosmology and because it has eluded all attempts at solution, is regarded by mathematicians as a holy grail.

Yau, a stocky man of fifty-seven, stood at a lectern in shirtsleeves and black-rimmed glasses and, with his hands in his pockets, described how two of his students, Xi-Ping Zhu and Huai-Dong Cao, had completed a proof of the Poincaré conjecture a few weeks earlier. "I'm very positive about Zhu and Cao's work," Yau said. "Chinese mathematicians should have every reason to be proud of such a big success in completely solving the puzzle." He said that Zhu and Cao were indebted to his longtime American collaborator Richard Hamilton, who deserved most of the credit for solving the Poincaré. He also mentioned Grigory Perelman, a Russian mathematician who, he acknowledged, had made an important contribution. Nevertheless, Yau said, "in Perelman's work, spectacular as it is, many key ideas of the proofs are sketched or outlined, and complete details are often missing." He added, "We would like to get Perelman to make comments. But Perelman resides in St. Petersburg and refuses to communicate with other people."

For ninety minutes, Yau discussed some of the technical details of his students' proof. When he was finished, no one asked any questions. That night, however, a Brazilian physicist posted a report of the lecture on his blog. "Looks like China soon will take the lead also in mathematics," he wrote.

Grigory Perelman is indeed reclusive. He left his job as a researcher at the Steklov Institute of Mathematics, in St. Petersburg, last December; he has few friends; and he lives with his mother in an apartment on the outskirts of the city. Although he had never granted an interview before, he was cordial and frank when we visited him, in late June, shortly after Yau's conference in Beijing, taking us on a long walking tour of the city. "I'm looking for some friends, and they don't have to be mathematicians," he said. The week before the conference, Perelman had spent

hours discussing the Poincaré conjecture with Sir John M. Ball, the fifty-eight-year-old president of the International Mathematical Union, the discipline's influential professional association. The meeting, which took place at a conference center in a stately mansion overlooking the Neva River, was highly unusual. At the end of May, a committee of nine prominent mathematicians had voted to award Perelman a Fields Medal for his work on the Poincaré, and Ball had gone to St. Petersburg to persuade him to accept the prize in a public ceremony at the I.M.U.'s quadrennial congress, in Madrid, on August 22nd.

The Fields Medal, like the Nobel Prize, grew, in part, out of a desire to elevate science above national animosities. German mathematicians were excluded from the first I.M.U. congress, in 1924, and, though the ban was lifted before the next one, the trauma it caused led, in 1936, to the establishment of the Fields, a prize intended to be "as purely international and impersonal as possible."

However, the Fields Medal, which is awarded every four years, to between two and four mathematicians, is supposed not only to reward past achievements but also to stimulate future research; for this reason, it is given only to mathematicians aged forty and younger. In recent decades, as the number of professional mathematicians has grown, the Fields Medal has become increasingly prestigious. Only forty-four medals have been awarded in nearly seventy years—including three for work closely related to the Poincaré conjecture—and no mathematician has ever refused the prize. Nevertheless, Perelman told Ball that he had no intention of accepting it. "I refuse," he said simply.

Over a period of eight months, beginning in November, 2002, Perelman posted a proof of the Poincaré on the Internet in three installments. Like a sonnet or an aria, a mathematical proof has a distinct form and set of conventions. It begins with axioms, or accepted truths, and employs a series of logical statements to arrive at a conclusion. If the logic is deemed to be watertight, then the result is a theorem. Unlike proof in law or science, which is based on evidence and therefore subject to qualification and revision, a proof of a theorem is definitive. Judgments about the accuracy of a proof are mediated by peer-reviewed journals; to insure fairness, reviewers are supposed to be carefully chosen by journal editors, and the identity of a scholar whose pa-per is under consideration is kept secret. Publication implies that a proof is complete, correct, and original.

By these standards, Perelman's proof was unorthodox. It was astonishingly brief for such an ambitious piece of work; logic sequences that could have been elaborated over many pages were often severely compressed. Moreover, the proof made no direct mention of the Poincaré and included many elegant results that were irrelevant to the central argument. But, four years later, at least two teams of experts had vetted the proof and had found no significant gaps or errors in it. A consensus was emerging in the math community: Perelman had solved the Poincaré. Even so, the proof's complexity—and Perelman's use of shorthand in making some of his most important claims—made it vulnerable to challenge. Few mathematicians had the expertise necessary to evaluate and defend it.

After giving a series of lectures on the proof in the United States in 2003, Perelman returned to St. Petersburg. Since then, although he had continued to answer queries about it by e-mail, he had had minimal contact with colleagues and, for reasons no one understood, had not tried to publish it. Still, there was little doubt that Perelman, who turned forty on June 13th, deserved a Fields Medal. As Ball planned the I.M.U.'s 2006 congress, he began to conceive of it as a historic event. More than three thousand mathematicians would be attending, and King Juan Carlos of Spain had agreed to preside over the awards ceremony. The I.M.U.'s newsletter predicted that the congress would be remembered as "the occasion when this conjecture became a theorem." Ball, determined to make sure that Perelman would be there, decided to go to St. Petersburg.

Ball wanted to keep his visit a secret—the names of Fields Medal recipients are announced officially at the awards ceremony—and the conference center where he met with Perelman was deserted. For ten hours over two days, he tried to persuade Perelman to agree to accept the prize. Perelman, a slender, balding man with a curly beard, bushy eyebrows, and blue-green eyes, listened politely. He had not spoken English for three years, but he fluently parried Ball's entreaties, at one point taking Ball on a long walk—one of Perelman's favorite activities. As he summed up the conversation two weeks later: "He proposed to me three alternatives: accept and come; accept and don't come, and we will send you the medal later; third, I don't accept the prize. From the very beginning, I told him I have chosen the third one." The Fields Medal held no interest for him, Perelman explained. "It was completely irrelevant for me," he said. "Everybody understood that if the proof is correct then no other recognition is needed."

P roofs of the Poincaré have been announced nearly every year since the conjecture was formulated, by Henri Poincaré, more than a hundred years ago. Poincaré was a cousin of Raymond Poincaré, the President of France during the First World War, and one of the most creative mathematicians of the nineteenth century. Slight,

myopic, and notoriously absent-minded, he conceived his famous problem in 1904, eight years before he died, and tucked it as an offhand question into the end of a sixty-five-page paper.

Poincaré didn't make much progress on proving the conjecture. "*Cette question nous entraînerait trop loin*" ("This question would take us too far"), he wrote. He was a founder of topology, also known as "rubber-sheet geometry," for its focus on the intrinsic properties of spaces. From a topologist's perspective, there is no difference between a bagel and a coffee cup with a handle. Each has a single hole and can be manipulated to resemble the other without being torn or cut. Poincaré used the term "manifold" to describe such an abstract topological space. The simplest possible two-dimensional manifold is the surface of a soccer ball, which, to a topologist, is a sphere—even when it is stomped on, stretched, or crumpled. The proof that an object is a so-called two-sphere, since it can take on any number of shapes, is that it is "simply connected," meaning that no holes puncture it. Unlike a soccer ball, a bagel is not a true sphere. If you tie a slipknot around a soccer ball, you can easily pull the slipknot closed by sliding it along the surface of the ball. But if you tie a slipknot around a bagel through the hole in its middle you cannot pull the slipknot closed without tearing the bagel.

Two-dimensional manifolds were well understood by the mid-nineteenth century. But it remained unclear whether what was true for two dimensions was also true for three. Poincaré proposed that all closed, simply connected, three-dimensional manifolds—those which lack holes and are of finite extent—were spheres. The conjecture was potentially important for scientists studying the largest known three-dimensional manifold: the universe. Proving it mathematically, however, was far from easy. Most attempts were merely embarrassing, but some led to important mathematical discoveries, including proofs of Dehn's Lemma, the Sphere Theorem, and the Loop Theorem, which are now fundamental concepts in topology.

By the nineteen-sixties, topology had become one of the most productive areas of mathematics, and young topologists were launching regular attacks on the Poincaré. To the astonishment of most mathematicians, it turned out that manifolds of the fourth, fifth, and higher dimensions were more tractable than those of the third dimension. By 1982, Poincaré's conjecture had been proved in all dimensions except the third. In 2000, the Clay Mathematics Institute, a private foundation that promotes mathematical research, named the Poincaré one of the seven most important outstanding problems in mathematics and offered a million dollars to anyone who could prove it.

"My whole life as a mathematician has been dominated by the Poincaré conjecture," John Morgan, the head of the mathematics department at Columbia University, said. "I never thought I'd see a solution. I thought nobody could touch it."

Grigory Perelman did not plan to become a mathematician. "There was never a decision point," he said when we met. We were outside the apartment building where he lives, in Kupchino, a neighborhood of drab high-rises. Perelman's father, who was an electrical engineer, encouraged his interest in math. "He gave me logical and other math problems to think about," Perelman said. "He got a lot of books for me to read. He taught me how to play chess. He was proud of me." Among the books his father gave him was a copy of "Physics for Entertainment," which had been a best-seller in the Soviet Union in the nineteen-thirties. In the foreword, the book's author describes the contents as "conundrums, brain-teasers, entertaining anecdotes, and unexpected comparisons," adding, "I have quoted extensively from Jules Verne, H. G. Wells, Mark Twain and other writers, because, besides providing entertainment, the fantastic experiments these writers describe may well serve as instructive illustrations at physics classes." The book's topics included how to jump from a moving car, and why, "according to the law of buoyancy, we would never drown in the Dead Sea."

The notion that Russian society considered worthwhile what Perelman did for pleasure came as a surprise. By the time he was fourteen, he was the star performer of a local math club. In 1982, the year that Shing-Tung Yau won a Fields Medal, Perelman earned a perfect score and the gold medal at the International Mathematical Olympiad, in Budapest. He was friendly with his teammates but not close—"I had no close friends," he said. He was one of two or three Jews in his grade, and he had a passion for opera, which also set him apart from his peers. His mother, a math teacher at a technical college, played the violin and began taking him to the opera when he was six. By the time Perelman was fifteen, he was spending his pocket money on records. He was thrilled to own a recording of a famous 1946 performance of "La Traviata," featuring Licia Albanese as Violetta. "Her voice was very good," he said.

At Leningrad University, which Perelman entered in 1982, at the age of sixteen, he took advanced classes in geometry and solved a problem posed by Yuri Burago, a mathematician at the Steklov Institute, who later became his Ph.D. adviser. "There are a lot of students of high ability who speak before thinking," Burago said. "Grisha was

different. He thought deeply. His answers were always correct. He always checked very, very carefully." Burago added, "He was not fast. Speed means nothing. Math doesn't depend on speed. It is about *deep*."

At the Steklov in the early nineties, Perelman became an expert on the geometry of Riemannian and Alexandrov spaces—extensions of traditional Euclidean geometry—and began to publish articles in the leading Russian and American mathematics journals. In 1992, Perelman was invited to spend a semester each at New York University and Stony Brook University. By the time he left for the United States, that fall, the Russian economy had collapsed. Dan Stroock, a mathematician at M.I.T., recalls smuggling wads of dollars into the country to deliver to a retired mathematician at the Steklov, who, like many of his colleagues, had become destitute.

Perelman was pleased to be in the United States, the capital of the international mathematics community. He wore the same brown corduroy jacket every day and told friends at N.Y.U. that he lived on a diet of bread, cheese, and milk. He liked to walk to Brooklyn, where he had relatives and could buy traditional Russian brown bread. Some of his colleagues were taken aback by his fingernails, which were several inches long. "If they grow, why wouldn't I let them grow?" he would say when someone asked why he didn't cut them. Once a week, he and a young Chinese mathematician named Gang Tian drove to Princeton, to attend a seminar at the Institute for Advanced Study.

For several decades, the institute and nearby Princeton University had been centers of topological research. In the late seventies, William Thurston, a Princeton mathematician who liked to test out his ideas using scissors and construction paper, proposed a taxonomy for classifying manifolds of three dimensions. He argued that, while the manifolds could be made to take on many different shapes, they nonetheless had a "preferred" geometry, just as a piece of silk draped over a dressmaker's mannequin takes on the mannequin's form.

Thurston proposed that every three-dimensional manifold could be broken down into one or more of eight types of component, including a spherical type. Thurston's theory—which became known as the geometrization conjecture—describes all possible three-dimensional manifolds and is thus a powerful generalization of the Poincaré. If it was confirmed, then Poincaré's conjecture would be, too. Proving Thurston and Poincaré "definitely swings open doors," Barry Mazur, a mathematician at Harvard, said. The implications of the conjectures for other disciplines may not be apparent for years, but for mathematicians the problems are fundamental. "This is a kind of twentieth-century Pythagorean theorem," Mazur added. "It changes the landscape."

In 1982, Thurston won a Fields Medal for his contributions to topology. That year, Richard Hamilton, a mathematician at Cornell, published a paper on an equation called the Ricci flow, which he suspected could be relevant for solving Thurston's conjecture and thus the Poincaré. Like a heat equation, which describes how heat distributes itself evenly through a substance—flowing from hotter to cooler parts of a metal sheet, for example—to create a more uniform temperature, the Ricci flow, by smoothing out irregularities, gives manifolds a more uniform geometry.

Hamilton, the son of a Cincinnati doctor, defied the math profession's nerdy stereotype. Brash and irreverent, he rode horses, windsurfed, and had a succession of girlfriends. He treated math as merely one of life's pleasures. At forty-nine, he was considered a brilliant lecturer, but he had published relatively little beyond a series of seminal articles on the Ricci flow, and he had few graduate students. Perelman had read Hamilton's papers and went to hear him give a talk at the Institute for Advanced Study. Afterward, Perelman shyly spoke to him.

"I really wanted to ask him something," Perelman recalled. "He was smiling, and he was quite patient. He actually told me a couple of things that he published a few years later. He did not hesitate to tell me. Hamilton's openness and generosity—it really attracted me. I can't say that most mathematicians act like that.

"I was working on different things, though occasionally I would think about the Ricci flow," Perelman added. "You didn't have to be a great mathematician to see that this would be useful for geometrization. I felt I didn't know very much. I kept asking questions."

S hing-Tung Yau was also asking Hamilton questions about the Ricci flow. Yau and Hamilton had met in the seventies, and had become close, despite considerable differences in temperament and background. A mathematician at the University of California at San Diego who knows both men called them "the mathematical loves of each other's lives."

Yau's family moved to Hong Kong from mainland China in 1949, when he was five months old, along with hundreds of thousands of other refugees fleeing Mao's armies. The previous year, his father, a relief worker for the United Nations, had lost most of the family's savings in a series of failed ventures. In Hong Kong, to support his wife and eight children, he tutored college students in classical Chinese literature and philosophy.

When Yau was fourteen, his father died of kidney cancer, leaving his mother dependent on handouts from Christian missionaries and whatever small sums she earned from selling handicrafts. Until then, Yau had been an indifferent student. But he began to devote himself to schoolwork, tutoring other students in math to make money. "Part of the thing that drives Yau is that he sees his own life as being his father's revenge," said Dan Stroock, the M.I.T. mathematician, who has known Yau for twenty years. "Yau's father was like the Talmudist whose children are starving."

Yau studied math at the Chinese University of Hong Kong, where he attracted the attention of Shiing-Shen Chern, the preëminent Chinese mathematician, who helped him win a scholarship to the University of California at Berkeley. Chern was the author of a famous theorem combining topology and geometry. He spent most of his career in the United States, at Berkeley. He made frequent visits to Hong Kong, Taiwan, and, later, China, where he was a revered symbol of Chinese intellectual achievement, to promote the study of math and science.

In 1969, Yau started graduate school at Berkeley, enrolling in seven graduate courses each term and auditing several others. He sent half of his scholarship money back to his mother in China and impressed his professors with his tenacity. He was obliged to share credit for his first major result when he learned that two other mathematicians were working on the same problem. In 1976, he proved a twenty-year-old conjecture pertaining to a type of manifold that is now crucial to string theory. A French mathematician had formulated a proof of the problem, which is known as Calabi's conjecture, but Yau's, because it was more general, was more powerful. (Physicists now refer to Calabi-Yau manifolds.) "He was not so much thinking up some original way of looking at a subject but solving extremely hard technical problems that at the time only he could solve, by sheer intellect and force of will," Phillip Griffiths, a geometer and a former director of the Institute for Advanced Study, said.

In 1980, when Yau was thirty, he became one of the youngest mathematicians ever to be appointed to the permanent faculty of the Institute for Advanced Study, and he began to attract talented students. He won a Fields Medal two years later, the first Chinese ever to do so. By this time, Chern was seventy years old and on the verge of retirement. According to a relative of Chern's, "Yau decided that he was going to be the next famous Chinese mathematician and that it was time for Chern to step down."

Harvard had been trying to recruit Yau, and when, in 1983, it was about to make him a second offer Phillip Griffiths told the dean of faculty a version of a story from "The Romance of the Three Kingdoms," a Chinese classic. In the third century A.D., a Chinese warlord dreamed of creating an empire, but the most brilliant general in China was working for a rival. Three times, the warlord went to his enemy's kingdom to seek out the general. Impressed, the general agreed to join him, and together they succeeded in founding a dynasty. Taking the hint, the dean flew to Philadelphia, where Yau lived at the time, to make him an offer. Even so, Yau turned down the job. Finally, in 1987, he agreed to go to Harvard.

Yau's entrepreneurial drive extended to collaborations with colleagues and students, and, in addition to conducting his own research, he began organizing seminars. He frequently allied himself with brilliantly inventive mathematicians, including Richard Schoen and William Meeks. But Yau was especially impressed by Hamilton, as much for his swagger as for his imagination. "I can have fun with Hamilton," Yau told us during the string-theory conference in Beijing. "I can go swimming with him. I go out with him and his girlfriends and all that." Yau was convinced that Hamilton could use the Ricci-flow equation to solve the Poincaré and Thurston conjectures, and he urged him to focus on the problems. "Meeting Yau changed his mathematical life," a friend of both mathematicians said of Hamilton. "This was the first time he had been on to something extremely big. Talking to Yau gave him courage and direction."

Yau believed that if he could help solve the Poincaré it would be a victory not just for him but also for China. In the mid-nineties, Yau and several other Chinese scholars began meeting with President Jiang Zemin to discuss how to rebuild the country's scientific institutions, which had been largely destroyed during the Cultural Revolution. Chinese universities were in dire condition. According to Steve Smale, who won a Fields for proving the Poincaré in higher dimensions, and who, after retiring from Berkeley, taught in Hong Kong, Peking University had "halls filled with the smell of urine, one common room, one office for all the assistant professors," and paid its faculty wretchedly low salaries. Yau persuaded a Hong Kong real-estate mogul to help finance a mathematics institute at the Chinese Academy of Sciences, in Beijing, and to endow a Fields-style medal for Chinese mathematicians under the age of forty-five. On his trips to China, Yau touted Hamilton and their joint work on the Ricci flow and the Poincaré as a model for young Chinese mathematicians. As he put it in Beijing, "They always say that the whole country should learn from Mao or some big heroes. So I made a joke to them, but I was half serious. I said the whole country should

learn from Hamilton.”

G rigory Perelman was learning from Hamilton already. In 1993, he began a two-year fellowship at Berkeley. While he was there, Hamilton gave several talks on campus, and in one he mentioned that he was working on the Poincaré. Hamilton’s Ricci-flow strategy was extremely technical and tricky to execute. After one of his talks at Berkeley, he told Perelman about his biggest obstacle. As a space is smoothed under the Ricci flow, some regions deform into what mathematicians refer to as “singularities.” Some regions, called “necks,” become attenuated areas of infinite density. More troubling to Hamilton was a kind of singularity he called the “cigar.” If cigars formed, Hamilton worried, it might be impossible to achieve uniform geometry. Perelman realized that a paper he had written on Alexandrov spaces might help Hamilton prove Thurston’s conjecture—and the Poincaré—once Hamilton solved the cigar problem. “At some point, I asked Hamilton if he knew a certain collapsing result that I had proved but not published—which turned out to be very useful,” Perelman said. “Later, I realized that he didn’t understand what I was talking about.” Dan Stroock, of M.I.T., said, “Perelman may have learned stuff from Yau and Hamilton, but, at the time, they were not learning from him.”

By the end of his first year at Berkeley, Perelman had written several strikingly original papers. He was asked to give a lecture at the 1994 I.M.U. congress, in Zurich, and invited to apply for jobs at Stanford, Princeton, the Institute for Advanced Study, and the University of Tel Aviv. Like Yau, Perelman was a formidable problem solver. Instead of spending years constructing an intricate theoretical framework, or defining new areas of research, he focussed on obtaining particular results. According to Mikhail Gromov, a renowned Russian geometer who has collaborated with Perelman, he had been trying to overcome a technical difficulty relating to Alexandrov spaces and had apparently been stumped. “He couldn’t do it,” Gromov said. “It was hopeless.”

Perelman told us that he liked to work on several problems at once. At Berkeley, however, he found himself returning again and again to Hamilton’s Ricci-flow equation and the problem that Hamilton thought he could solve with it. Some of Perelman’s friends noticed that he was becoming more and more ascetic. Visitors from St. Petersburg who stayed in his apartment were struck by how sparsely furnished it was. Others worried that he seemed to want to reduce life to a set of rigid axioms. When a member of a hiring committee at Stanford asked him for a C.V. to include with requests for letters of recommendation, Perelman balked. “If they know my work, they don’t need my C.V.,” he said. “If they need my C.V., they don’t know my work.”

Ultimately, he received several job offers. But he declined them all, and in the summer of 1995 returned to St. Petersburg, to his old job at the Steklov Institute, where he was paid less than a hundred dollars a month. (He told a friend that he had saved enough money in the United States to live on for the rest of his life.) His father had moved to Israel two years earlier, and his younger sister was planning to join him there after she finished college. His mother, however, had decided to remain in St. Petersburg, and Perelman moved in with her. “I realize that in Russia I work better,” he told colleagues at the Steklov.

At twenty-nine, Perelman was firmly established as a mathematician and yet largely unburdened by professional responsibilities. He was free to pursue whatever problems he wanted to, and he knew that his work, should he choose to publish it, would be shown serious consideration. Yakov Eliashberg, a mathematician at Stanford who knew Perelman at Berkeley, thinks that Perelman returned to Russia in order to work on the Poincaré. “Why not?” Perelman said when we asked whether Eliashberg’s hunch was correct.

The Internet made it possible for Perelman to work alone while continuing to tap a common pool of knowledge. Perelman searched Hamilton’s papers for clues to his thinking and gave several seminars on his work. “He didn’t need any help,” Gromov said. “He likes to be alone. He reminds me of Newton—this obsession with an idea, working by yourself, the disregard for other people’s opinion. Newton was more obnoxious. Perelman is nicer, but very obsessed.”

In 1995, Hamilton published a paper in which he discussed a few of his ideas for completing a proof of the Poincaré. Reading the paper, Perelman realized that Hamilton had made no progress on overcoming his obstacles—the necks and the cigars. “I hadn’t seen any evidence of progress after early 1992,” Perelman told us. “Maybe he got stuck even earlier.” However, Perelman thought he saw a way around the impasse. In 1996, he wrote Hamilton a long letter outlining his notion, in the hope of collaborating. “He did not answer,” Perelman said. “So I decided to work alone.”

Y au had no idea that Hamilton’s work on the Poincaré had stalled. He was increasingly anxious about his own

standing in the mathematics profession, particularly in China, where, he worried, a younger scholar could try to supplant him as Chern's heir. More than a decade had passed since Yau had proved his last major result, though he continued to publish prolifically. "Yau wants to be the king of geometry," Michael Anderson, a geometer at Stony Brook, said. "He believes that everything should issue from him, that he should have oversight. He doesn't like people encroaching on his territory." Determined to retain control over his field, Yau pushed his students to tackle big problems. At Harvard, he ran a notoriously tough seminar on differential geometry, which met for three hours at a time three times a week. Each student was assigned a recently published proof and asked to reconstruct it, fixing any errors and filling in gaps. Yau believed that a mathematician has an obligation to be explicit, and impressed on his students the importance of step-by-step rigor.

There are two ways to get credit for an original contribution in mathematics. The first is to produce an original proof. The second is to identify a significant gap in someone else's proof and supply the missing chunk. However, only true mathematical gaps—missing or mistaken arguments—can be the basis for a claim of originality. Filling in gaps in exposition—shortcuts and abbreviations used to make a proof more efficient—does not count. When, in 1993, Andrew Wiles revealed that a gap had been found in his proof of Fermat's last theorem, the problem became fair game for anyone, until, the following year, Wiles fixed the error. Most mathematicians would agree that, by contrast, if a proof's implicit steps can be made explicit by an expert, then the gap is merely one of exposition, and the proof should be considered complete and correct.

Occasionally, the difference between a mathematical gap and a gap in exposition can be hard to discern. On at least one occasion, Yau and his students have seemed to confuse the two, making claims of originality that other mathematicians believe are unwarranted. In 1996, a young geometer at Berkeley named Alexander Givental had proved a mathematical conjecture about mirror symmetry, a concept that is fundamental to string theory. Though other mathematicians found Givental's proof hard to follow, they were optimistic that he had solved the problem. As one geometer put it, "Nobody at the time said it was incomplete and incorrect."

In the fall of 1997, Kefeng Liu, a former student of Yau's who taught at Stanford, gave a talk at Harvard on mirror symmetry. According to two geometers in the audience, Liu proceeded to present a proof strikingly similar to Givental's, describing it as a paper that he had co-authored with Yau and another student of Yau's. "Liu mentioned Givental but only as one of a long list of people who had contributed to the field," one of the geometers said. (Liu maintains that his proof was significantly different from Givental's.)

Around the same time, Givental received an e-mail signed by Yau and his collaborators, explaining that they had found his arguments impossible to follow and his notation baffling, and had come up with a proof of their own. They praised Givental for his "brilliant idea" and wrote, "In the final version of our paper your important contribution will be acknowledged."

A few weeks later, the paper, "Mirror Principle I," appeared in the *Asian Journal of Mathematics*, which is co-edited by Yau. In it, Yau and his coauthors describe their result as "the first complete proof" of the mirror conjecture. They mention Givental's work only in passing. "Unfortunately," they write, his proof, "which has been read by many prominent experts, is incomplete." However, they did not identify a specific mathematical gap.

Givental was taken aback. "I wanted to know what their objection was," he told us. "Not to expose them or defend myself." In March, 1998, he published a paper that included a three-page footnote in which he pointed out a number of similarities between Yau's proof and his own. Several months later, a young mathematician at the University of Chicago who was asked by senior colleagues to investigate the dispute concluded that Givental's proof was complete. Yau says that he had been working on the proof for years with his students and that they achieved their result independently of Givental. "We had our own ideas, and we wrote them up," he says.

Around this time, Yau had his first serious conflict with Chern and the Chinese mathematical establishment. For years, Chern had been hoping to bring the I.M.U.'s congress to Beijing. According to several mathematicians who were active in the I.M.U. at the time, Yau made an eleventh-hour effort to have the congress take place in Hong Kong instead. But he failed to persuade a sufficient number of colleagues to go along with his proposal, and the I.M.U. ultimately decided to hold the 2002 congress in Beijing. (Yau denies that he tried to bring the congress to Hong Kong.) Among the delegates the I.M.U. appointed to a group that would be choosing speakers for the congress was Yau's most successful student, Gang Tian, who had been at N.Y.U. with Perelman and was now a professor at M.I.T. The host committee in Beijing also asked Tian to give a plenary address.

Yau was caught by surprise. In March, 2000, he had published a survey of recent research in his field studded with glowing references to Tian and to their joint projects. He retaliated by organizing his first conference on string

theory, which opened in Beijing a few days before the math congress began, in late August, 2002. He persuaded Stephen Hawking and several Nobel laureates to attend, and for days the Chinese newspapers were full of pictures of famous scientists. Yau even managed to arrange for his group to have an audience with Jiang Zemin. A mathematician who helped organize the math congress recalls that along the highway between Beijing and the airport there were "billboards with pictures of Stephen Hawking plastered everywhere."

That summer, Yau wasn't thinking much about the Poincaré. He had confidence in Hamilton, despite his slow pace. "Hamilton is a very good friend," Yau told us in Beijing. "He is more than a friend. He is a hero. He is so original. We were working to finish our proof. Hamilton worked on it for twenty-five years. You work, you get tired. He probably got a little tired—and you want to take a rest."

Then, on November 12, 2002, Yau received an e-mail message from a Russian mathematician whose name didn't immediately register. "May I bring to your attention my paper," the e-mail said.

On November 11th, Perelman had posted a thirty-nine-page paper entitled "The Entropy Formula for the Ricci Flow and Its Geometric Applications," on arXiv.org, a Web site used by mathematicians to post preprints—articles awaiting publication in refereed journals. He then e-mailed an abstract of his paper to a dozen mathematicians in the United States—including Hamilton, Tian, and Yau—none of whom had heard from him for years. In the abstract, he explained that he had written "a sketch of an eclectic proof" of the geometrization conjecture.

Perelman had not mentioned the proof or shown it to anyone. "I didn't have any friends with whom I could discuss this," he said in St. Petersburg. "I didn't want to discuss my work with someone I didn't trust." Andrew Wiles had also kept the fact that he was working on Fermat's last theorem a secret, but he had had a colleague vet the proof before making it public. Perelman, by casually posting a proof on the Internet of one of the most famous problems in mathematics, was not just flouting academic convention but taking a considerable risk. If the proof was flawed, he would be publicly humiliated, and there would be no way to prevent another mathematician from fixing any errors and claiming victory. But Perelman said he was not particularly concerned. "My reasoning was: if I made an error and someone used my work to construct a correct proof I would be pleased," he said. "I never set out to be the sole solver of the Poincaré."

Gang Tian was in his office at M.I.T. when he received Perelman's e-mail. He and Perelman had been friendly in 1992, when they were both at N.Y.U. and had attended the same weekly math seminar in Princeton. "I immediately realized its importance," Tian said of Perelman's paper. Tian began to read the paper and discuss it with colleagues, who were equally enthusiastic.

On November 19th, Vitali Kapovitch, a geometer, sent Perelman an e-mail:

Hi Grisha, Sorry to bother you but a lot of people are asking me about your preprint "The entropy formula for the Ricci …" Do I understand it correctly that while you cannot yet do all the steps in the Hamilton program you can do enough so that using some collapsing results you can prove geometrization? Vitali.

Perelman's response, the next day, was terse: "That's correct. Grisha."

In fact, what Perelman had posted on the Internet was only the first installment of his proof. But it was sufficient for mathematicians to see that he had figured out how to solve the Poincaré. Barry Mazur, the Harvard mathematician, uses the image of a dented fender to describe Perelman's achievement: "Suppose your car has a dented fender and you call a mechanic to ask how to smooth it out. The mechanic would have a hard time telling you what to do over the phone. You would have to bring the car into the garage for him to examine. Then he could tell you where to give it a few knocks. What Hamilton introduced and Perelman completed is a procedure that is independent of the particularities of the blemish. If you apply the Ricci flow to a 3-D space, it will begin to undent it and smooth it out. The mechanic would not need to even see the car—just apply the equation." Perelman proved that the "cigars" that had troubled Hamilton could not actually occur, and he showed that the "neck" problem could be solved by performing an intricate sequence of mathematical surgeries: cutting out singularities and patching up the raw edges. "Now we have a procedure to smooth things and, at crucial points, control the breaks," Mazur said.

Tian wrote to Perelman, asking him to lecture on his paper at M.I.T. Colleagues at Princeton and Stony Brook extended similar invitations. Perelman accepted them all and was booked for a month of lectures beginning in April, 2003. "Why not?" he told us with a shrug. Speaking of mathematicians generally, Fedor Nazarov, a mathematician at Michigan State University, said, "After you've solved a problem, you have a great urge to talk about it."

Hamilton and Yau were stunned by Perelman's announcement. "We felt that nobody else would be able to

discover the solution," Yau told us in Beijing. "But then, in 2002, Perelman said that he published something. He basically did a shortcut without doing all the detailed estimates that we did." Moreover, Yau complained, Perelman's proof "was written in such a messy way that we didn't understand."

Perelman's April lecture tour was treated by mathematicians and by the press as a major event. Among the audience at his talk at Princeton were John Ball, Andrew Wiles, John Forbes Nash, Jr., who had proved the Riemannian embedding theorem, and John Conway, the inventor of the cellular automaton game Life. To the astonishment of many in the audience, Perelman said nothing about the Poincaré. "Here is a guy who proved a world-famous theorem and didn't even mention it," Frank Quinn, a mathematician at Virginia Tech, said. "He stated some key points and special properties, and then answered questions. He was establishing credibility. If he had beaten his chest and said, 'I solved it,' he would have got a huge amount of resistance." He added, "People were expecting a strange sight. Perelman was much more normal than they expected."

To Perelman's disappointment, Hamilton did not attend that lecture or the next ones, at Stony Brook. "I'm a disciple of Hamilton's, though I haven't received his authorization," Perelman told us. But John Morgan, at Columbia, where Hamilton now taught, was in the audience at Stony Brook, and after a lecture he invited Perelman to speak at Columbia. Perelman, hoping to see Hamilton, agreed. The lecture took place on a Saturday morning. Hamilton showed up late and asked no questions during either the long discussion session that followed the talk or the lunch after that. "I had the impression he had read only the first part of my paper," Perelman said.

In the April 18, 2003, issue of *Science*, Yau was featured in an article about Perelman's proof: "Many experts, although not all, seem convinced that Perelman has stubbed out the cigars and tamed the narrow necks. But they are less confident that he can control the number of surgeries. That could prove a fatal flaw, Yau warns, noting that many other attempted proofs of the Poincaré conjecture have stumbled over similar missing steps." Proofs should be treated with skepticism until mathematicians have had a chance to review them thoroughly, Yau told us. Until then, he said, "it's not math—it's religion."

By mid-July, Perelman had posted the final two installments of his proof on the Internet, and mathematicians had begun the work of formal explication, painstakingly retracing his steps. In the United States, at least two teams of experts had assigned themselves this task: Gang Tian (Yau's rival) and John Morgan; and a pair of researchers at the University of Michigan. Both projects were supported by the Clay Institute, which planned to publish Tian and Morgan's work as a book. The book, in addition to providing other mathematicians with a guide to Perelman's logic, would allow him to be considered for the Clay Institute's million-dollar prize for solving the Poincaré. (To be eligible, a proof must be published in a peer-reviewed venue and withstand two years of scrutiny by the mathematical community.)

On September 10, 2004, more than a year after Perelman returned to St. Petersburg, he received a long e-mail from Tian, who said that he had just attended a two-week workshop at Princeton devoted to Perelman's proof. "I think that we have understood the whole paper," Tian wrote. "It is all right."

Perelman did not write back. As he explained to us, "I didn't worry too much myself. This was a famous problem. Some people needed time to get accustomed to the fact that this is no longer a conjecture. I personally decided for myself that it was right for me to stay away from verification and not to participate in all these meetings. It is important for me that I don't influence this process."

In July of that year, the National Science Foundation had given nearly a million dollars in grants to Yau, Hamilton, and several students of Yau's to study and apply Perelman's "breakthrough." An entire branch of mathematics had grown up around efforts to solve the Poincaré, and now that branch appeared at risk of becoming obsolete. Michael Freedman, who won a Fields for proving the Poincaré conjecture for the fourth dimension, told the *Times* that Perelman's proof was a "small sorrow for this particular branch of topology." Yuri Burago said, "It kills the field. After this is done, many mathematicians will move to other branches of mathematics."

Five months later, Chern died, and Yau's efforts to insure that he-—not Tian—was recognized as his successor turned vicious. "It's all about their primacy in China and their leadership among the expatriate Chinese," Joseph Kohn, a former chairman of the Prince-ton mathematics department, said. "Yau's not jealous of Tian's mathematics, but he's jealous of his power back in China."

Though Yau had not spent more than a few months at a time on mainland China since he was an infant, he was convinced that his status as the only Chinese Fields Medal winner should make him Chern's successor. In a speech he gave at Zhejiang University, in Hangzhou, during the summer of 2004, Yau reminded his listeners of his Chinese

roots. "When I stepped out from the airplane, I touched the soil of Beijing and felt great joy to be in my mother country," he said. "I am proud to say that when I was awarded the Fields Medal in mathematics, I held no passport of any country and should certainly be considered Chinese."

The following summer, Yau returned to China and, in a series of interviews with Chinese reporters, attacked Tian and the mathematicians at Peking University. In an article published in a Beijing science newspaper, which ran under the headline "SHING-TUNG YAU IS SLAMMING ACADEMIC CORRUPTION IN CHINA," Yau called Tian "a complete mess." He accused him of holding multiple professorships and of collecting a hundred and twenty-five thousand dollars for a few months' work at a Chinese university, while students were living on a hundred dollars a month. He also charged Tian with shoddy scholarship and plagiarism, and with intimidating his graduate students into letting him add his name to their papers. "Since I promoted him all the way to his academic fame today, I should also take responsibility for his improper behavior," Yau was quoted as saying to a reporter, explaining why he felt obliged to speak out.

In another interview, Yau described how the Fields committee had passed Tian over in 1988 and how he had lobbied on Tian's behalf with various prize committees, including one at the National Science Foundation, which awarded Tian five hundred thousand dollars in 1994.

Tian was appalled by Yau's attacks, but he felt that, as Yau's former student, there was little he could do about them. "His accusations were baseless," Tian told us. But, he added, "I have deep roots in Chinese culture. A teacher is a teacher. There is respect. It is very hard for me to think of anything to do."

While Yau was in China, he visited Xi-Ping Zhu, a protégé of his who was now chairman of the mathematics department at Sun Yat-sen University. In the spring of 2003, after Perelman completed his lecture tour in the United States, Yau had recruited Zhu and another student, Huai-Dong Cao, a professor at Lehigh University, to undertake an explication of Perelman's proof. Zhu and Cao had studied the Ricci flow under Yau, who considered Zhu, in particular, to be a mathematician of exceptional promise. "We have to figure out whether Perelman's paper holds together," Yau told them. Yau arranged for Zhu to spend the 2005-06 academic year at Harvard, where he gave a seminar on Perelman's proof and continued to work on his paper with Cao.

O n April 13th of this year, the thirty-one mathematicians on the editorial board of the *Asian Journal of Mathematics* received a brief e-mail from Yau and the journal's co-editor informing them that they had three days to comment on a paper by Xi-Ping Zhu and Huai-Dong Cao titled "The Hamilton-Perelman Theory of Ricci Flow: The Poincaré and Geometrization Conjectures," which Yau planned to publish in the journal. The e-mail did not include a copy of the paper, reports from referees, or an abstract. At least one board member asked to see the paper but was told that it was not available. On April 16th, Cao received a message from Yau telling him that the paper had been accepted by the *A.J.M.*, and an abstract was posted on the journal's Web site.

A month later, Yau had lunch in Cambridge with Jim Carlson, the president of the Clay Institute. He told Carlson that he wanted to trade a copy of Zhu and Cao's paper for a copy of Tian and Morgan's book manuscript. Yau told us he was worried that Tian would try to steal from Zhu and Cao's work, and he wanted to give each party simultaneous access to what the other had written. "I had a lunch with Carlson to request to exchange both manuscripts to make sure that nobody can copy the other," Yau said. Carlson demurred, explaining that the Clay Institute had not yet received Tian and Morgan's complete manuscript.

By the end of the following week, the title of Zhu and Cao's paper on the *A.J.M.'s* Web site had changed, to "A Complete Proof of the Poincaré and Geometrization Conjectures: Application of the Hamilton-Perelman Theory of the Ricci Flow." The abstract had also been revised. A new sentence explained, "This proof should be considered as the crowning achievement of the Hamilton-Perelman theory of Ricci flow."

Zhu and Cao's paper was more than three hundred pages long and filled the *A.J.M.'s* entire June issue. The bulk of the paper is devoted to reconstructing many of Hamilton's Ricci-flow results—including results that Perelman had made use of in his proof—and much of Perelman's proof of the Poincaré. In their introduction, Zhu and Cao credit Perelman with having "brought in fresh new ideas to figure out important steps to overcome the main obstacles that remained in the program of Hamilton." However, they write, they were obliged to "substitute several key arguments of Perelman by new approaches based on our study, because we were unable to comprehend these original arguments of Perelman which are essential to the completion of the geometrization program." Mathematicians familiar with Perelman's proof disputed the idea that Zhu and Cao had contributed significant new approaches to the Poincaré. "Perelman already did it and what he did was complete and correct," John Morgan said. "I don't see that they did anything different."

By early June, Yau had begun to promote the proof publicly. On June 3rd, at his mathematics institute in Beijing, he held a press conference. The acting director of the mathematics institute, attempting to explain the relative contributions of the different mathematicians who had worked on the Poincaré, said, "Hamilton contributed over fifty per cent; the Russian, Perelman, about twenty-five per cent; and the Chinese, Yau, Zhu, and Cao et al., about thirty per cent." (Evidently, simple addition can sometimes trip up even a mathematician.) Yau added, "Given the significance of the Poincaré, that Chinese mathematicians played a thirty-per-cent role is by no means easy. It is a very important contribution."

On June 12th, the week before Yau's conference on string theory opened in Beijing, the *South China Morning Post* reported, "Mainland mathematicians who helped crack a 'millennium math problem' will present the methodology and findings to physicist Stephen Hawking… . Yau Shing-Tung, who organized Professor Hawking's visit and is also Professor Cao's teacher, said yesterday he would present the findings to Professor Hawking because he believed the knowledge would help his research into the formation of black holes."

On the morning of his lecture in Beijing, Yau told us, "We want our contribution understood. And this is also a strategy to encourage Zhu, who is in China and who has done really spectacular work. I mean, important work with a century-long problem, which will probably have another few century-long implications. If you can attach your name in any way, it is a contribution."

E. T. Bell, the author of "Men of Mathematics," a witty history of the discipline published in 1937, once lamented "the squabbles over priority which disfigure scientific history." But in the days before e-mail, blogs, and Web sites, a certain decorum usually prevailed. In 1881, Poincaré, who was then at the University of Caen, had an altercation with a German mathematician in Leipzig named Felix Klein. Poincaré had published several papers in which he labelled certain functions "Fuchsian," after another mathematician. Klein wrote to Poincaré, pointing out that he and others had done significant work on these functions, too. An exchange of polite letters between Leipzig and Caen ensued. Poincaré's last word on the subject was a quote from Goethe's "Faust": "*Name ist Schall und Rauch.*" Loosely translated, that corresponds to Shakespeare's "What's in a name?"

This, essentially, is what Yau's friends are asking themselves. "I find myself getting annoyed with Yau that he seems to feel the need for more kudos," Dan Stroock, of M.I.T., said. "This is a guy who did magnificent things, for which he was magnificently rewarded. He won every prize to be won. I find it a little mean of him to seem to be trying to get a share of this as well." Stroock pointed out that, twenty-five years ago, Yau was in a situation very similar to the one Perelman is in today. His most famous result, on Calabi-Yau manifolds, was hugely important for theoretical physics. "Calabi outlined a program," Stroock said. "In a real sense, Yau was Calabi's Perelman. Now he's on the other side. He's had no compunction at all in taking the lion's share of credit for Calabi-Yau. And now he seems to be resenting Perelman getting credit for completing Hamilton's program. I don't know if the analogy has ever occurred to him."

Mathematics, more than many other fields, depends on collaboration. Most problems require the insights of several mathematicians in order to be solved, and the profession has evolved a standard for crediting individual contributions that is as stringent as the rules governing math itself. As Perelman put it, "If everyone is honest, it is natural to share ideas." Many mathematicians view Yau's conduct over the Poincaré as a violation of this basic ethic, and worry about the damage it has caused the profession. "Politics, power, and control have no legitimate role in our community, and they threaten the integrity of our field," Phillip Griffiths said.

Perelman likes to attend opera performances at the Mariinsky Theatre, in St. Petersburg. Sitting high up in the back of the house, he can't make out the singers' expressions or see the details of their costumes. But he cares only about the sound of their voices, and he says that the acoustics are better where he sits than anywhere else in the theatre. Perelman views the mathematics community—and much of the larger world—from a similar remove.

Before we arrived in St. Petersburg, on June 23rd, we had sent several messages to his e-mail address at the Steklov Institute, hoping to arrange a meeting, but he had not replied. We took a taxi to his apartment building and, reluctant to intrude on his privacy, left a book—a collection of John Nash's papers—in his mailbox, along with a card saying that we would be sitting on a bench in a nearby playground the following afternoon. The next day, after Perelman failed to appear, we left a box of pearl tea and a note describing some of the questions we hoped to discuss with him. We repeated this ritual a third time. Finally, believing that Perelman was out of town, we pressed the buzzer for his apartment, hoping at least to speak with his mother. A woman answered and let us inside. Perelman

met us in the dimly lit hallway of the apartment. It turned out that he had not checked his Steklov e-mail address for months, and had not looked in his mailbox all week. He had no idea who we were.

We arranged to meet at ten the following morning on Nevsky Prospekt. From there, Perelman, dressed in a sports coat and loafers, took us on a four-hour walking tour of the city, commenting on every building and vista. After that, we all went to a vocal competition at the St. Petersburg Conservatory, which lasted for five hours. Perelman repeatedly said that he had retired from the mathematics community and no longer considered himself a professional mathematician. He mentioned a dispute that he had had years earlier with a collaborator over how to credit the author of a particular proof, and said that he was dismayed by the discipline's lax ethics. "It is not people who break ethical standards who are regarded as aliens," he said. "It is people like me who are isolated." We asked him whether he had read Cao and Zhu's paper. "It is not clear to me what new contribution did they make," he said. "Apparently, Zhu did not quite understand the argument and reworked it." As for Yau, Perelman said, "I can't say I'm outraged. Other people do worse. Of course, there are many mathematicians who are more or less honest. But almost all of them are conformists. They are more or less honest, but they tolerate those who are not honest."

The prospect of being awarded a Fields Medal had forced him to make a complete break with his profession. "As long as I was not conspicuous, I had a choice," Perelman explained. "Either to make some ugly thing"—a fuss about the math community's lack of integrity—"or, if I didn't do this kind of thing, to be treated as a pet. Now, when I become a very conspicuous person, I cannot stay a pet and say nothing. That is why I had to quit." We asked Perelman whether, by refusing the Fields and withdrawing from his profession, he was eliminating any possibility of influencing the discipline. "I am not a politician!" he replied, angrily. Perelman would not say whether his objection to awards extended to the Clay Institute's million-dollar prize. "I'm not going to decide whether to accept the prize until it is offered," he said.

Mikhail Gromov, the Russian geometer, said that he understood Perelman's logic: "To do great work, you have to have a pure mind. You can think only about the mathematics. Everything else is human weakness. Accepting prizes is showing weakness." Others might view Perelman's refusal to accept a Fields as arrogant, Gromov said, but his principles are admirable. "The ideal scientist does science and cares about nothing else," he said. "He wants to live this ideal. Now, I don't think he really lives on this ideal plane. But he wants to." ♦