Award of the Millennium Prize to Grigoriy Perelman For Resolution of the Poincaré Conjecture

Laudations

Paris, June 8, 2010

Andrew Wiles

I would like to repeat Jim's welcome to Landon and Lavinia Clay, to the entire Clay family, to members of the Poincaré family and to all the mathematicians assembled here at this historic occasion. At its meeting ten years ago here in Paris, the Clay Mathematics Institute announced the establishment of prizes for seven of the world's most outstanding mathematical problems. The idea was not to give incentives to mathematicians to work harder. We do not need that. Nor was it to make mathematics more exciting for mathematicians. That too was unnecessary. Rather it was to record for posterity the great unsolved problems left over from the twentieth century. It served also to make mathematics more exciting for the non-mathematical world. In particular, it served and it still serves as a beacon to attract young people, children and students, to the ongoing excitement of mathematics and mathematical problems.

We made a conscious choice, in contrast to Hilbert, to pick problems that were already formulated and had stood the test of time. Of course, one big potential drawback might have been that the problems would rest for a long time unsolved. Then mathematics might appear static and the outside world might lose interest. The time frame for the solution of the great problems of antiquity, such as the squaring of the circle and the trisection of an angle was measured not in tens or even hundreds of years, but rather in thousands of years. It has therefore been a wonderful and perhaps unexpected surprise that we are able to be here today to celebrate the first of the Clay problems to be solved. This is especially true for this particular problem, which together with the Riemann hypothesis, has been on everyone's top list of mathematical problems for all of our lifetimes. For this we can thank Perelman, as well as his many predecessors, including particularly Hamilton.

Michael Atiyah

Famous Theorems define the mathematical landscape. They beckon from afar, rising dimly in the mists, an elusive challenge to the mathematical community. Are they accessible or is there some vast ravine or raging torrent that has to be traversed ?

When, after many years of exploration, pioneers reach the foothills, the ascent looks formidable or impossible. Initial attempts lead to dead-ends yor a return to the starting point. Explorations pack up and go home and prepare to tackle lesser heights.

But some mountaineers do not give up. They examine every aspect of the climb, spending many years identifying the optimal route. Then the final assault begins. Step by step the paths are cleared, the rivers crossed and the cliffs scaled. Finally on a glorious day the summit is reached, the mountain is tamed and a magnificent vista is opened up.

Today we celebrate such an event. A century after the death of Henri Poincaré, and in the city where he lived and worked, the Conjecture which he bequeathed to us has been settled. Grigory Perelman is the mountaineer who reached this pinnacle of the 3-dimensional world.

Exploration can be an end in itself but it also offers the land for other developments. Crops can be grown, cities built and art can flourish. Perelman has provided geometers with a fruitful land to cultivate.

William Thurston

It is a very special pleasure for me to have this occasion to publicly express my deep admiration and appreciation for Grigori Perelman.

Over several years in the 1970's, I developed to a vision of 3-dimensional manifolds as fitting into a beautiful geometric pattern, the geometrization conjecture, that became the central focus of my life's work. At a symposium on Poincaré in 1980, I felt emboldened to say that the geometrization conjecture put the Poincaré conjecture into a fuller and more constructive context. I expressed confidence that the geometrization conjecture is true, and I predicted it would be proven, but whether in one year or 100 years I could not say – I hoped it would be within my lifetime. I tried hard to prove it. I am truly gratified to see my hope finally become reality.

Perelman, with tremendous focus and virtuosity, constructed a beautiful proof where I and others failed. It is a proof that I could not have done: some of Perelman's strengths are my weaknesses. That the geometrization conjecture is true is not a surprise. That a proof like Perelman's could be valid is not a surprise: it has a certain rightness and inevitability, long dreamed of by many people (including me). What is surprising, wonderful and amazing is that someone – Perelman – succeeded in rigorously analyzing and controlling this process, despite the many hurdles, challenges and potential pitfalls. His method begin with a 3-dimensional shape that is irregular, complicated and hard to analyze or take in. The shape changes and evolves much like a bubble to even itself out, quickly smoothing small-scale irregularities, following the Ricci flow as developed by Richard Hamilton. Bubbles can pop: sometimes a bubble breaks up and splits apart, but Perelman found ways to analyze and control this process, to show that eventually all bubbles glide into a perfect form. Perelman's accomplishment gives us a solid foundation to build higher levels of understanding.

Perelman's aversion to public spectacle and to riches is mystifying to many. I have not talked to him about it and I can certainly not speak for him, but I want to say I have complete empathy and admiration for his inner strength and clarity, to be able to know and hold true to himself. Our true needs are deeper – -yet in our modern society most of us reflexively and relentlessly pursue wealth, consumer goods and admiration. We have learned from Perelman's mathematics. Perhaps we should also pause to reflect on ourselves and learn from Perelman's attitude toward life.

Simon Donaldson

It is no mere convention to say that it a great honour to be asked to speak here about the work of Grigory Perelman. From the time when his preprints concerning the Poincaré and Geometrisation Conjectures appeared, mathematicians around the world have been united in expressing their appreciation, awe and wonder at his extraordinary achievement, and I believe that I speak here as a representative of our whole intellectual community.

There are many signal qualities of Perelman's work. First, of course, it solves an outstanding, century-old, problem: a problem that has done much to drive the development of topology from its inception. Second, the work is, to the highest degree, original and profound. He introduced an entirely new idea which cut the Gordian knot that had held up the Ricci-flow approach – -

bearing on the central question of "collapsing" in Riemannian geometry. But that was just the beginning – -Perelman developed a host of extremely subtle and novel arguments: blending partial differential equations, differential geometry and the theory of convergence of spaces. The whole edifice he created, in his proof, is something unmatched, in its scope and depth, in this general area of mathematics. The ideas and techniques will have ramifications in many other problems for years to come. Third, there is a unique and romantic quality of his work. In modern mathematics, as in other endeavours, much progress is collaborative; either in the literal sense or, more generally, in developments driven by resonance between the ideas of different workers. Perelman's achievement is a testament to the continued power of the individual human mind in bringing about the most fundamental advances in mathematics.

Mikhail Gromov

The great 19th century mathematician Niels Henrik Abel wrote in his memoirs that in order to solve a difficult problem one has to to correctly formulate it.

A correct formulation is, usually, not as transparant and elementary as the original one – its significance is seen only a posteriori when the problem is eventually solved.

For example, the great achivement of 20th century mathematics – the proof of the "elementary" Fermat Last Theorem was obtained via a reduction to the Taniyama-Shimura-Weil conjecture, the very formulation of which relies on a profound "non-elementary" structure underlying the naive concept of an algebraic equation between integers. The solution of this conjecture by Andrew Wiles provided an understanding of this structure uncomparably deeper than the original message carried by the Fermat Theorem.

Similarly, Perelman has done much more than proving the Poincaré conjecture or even the more comprehensive Thurston's geometrization conjecture. What Perelman has achieved is the fulfillment of Richard Hamilton's program, thus revealing a profound structure in the space of 3-manifolds.

To get, by way of analogy, an idea of what Perelman accomplished, imagine that you have no overall picture of the geography of the Earth. You send one after another sea expedition in which discover new lands. Eventually, six continents are discovered. You keep sending hundreds of new expeditions, but they find nothing besides these six. You conjecture that there are no other great land masses on Earth. This is what Poincaré and Thurston say about the world of 3-manifolds: there are no manifolds besides those which has been already discovered.

Perelman's theorem goes far beyond proving this "non-existence" claim, just as Wiles' theorem tells you much more than non-existence of integer solutions of certain equations.

Perelman's work revealed the laws of Hamilton's flow – the “3D-plate tectonics” which shaped the world of 3-manifolds. Perelman then reconstructed the geography of 3D-world from these laws.

It will probably take a decade or so for the mathematical community to build up new edifices on the land discovered by Perelman.